1. INTRODUCTION

The ultimate goal of virtually all effort spent on modeling a petroleum field is to devise an optimal strategy to develop, manage, and operate the field. For some petroleum fields, optimization of production operations can be a major factor in increasing production rates and reducing production costs. While for single wells or other small systems simple nodal analysis may be adequate, large complex systems demand a much more sophisticated approach to predict the response of a large complicated production system accurately and to examine alternative operational scenarios efficiently. As optimization algorithms and reservoir simulation techniques continue to develop and computing power continues to increase, upstream oil and gas facilities previously thought not to be candidates for advanced control or optimization are being given new considerations (Clay et al., 1998). The objective of this study was to develop optimization methods for solving a class of important production operation problems for petroleum fields. In petroleum fields, hydrocarbon production is often constrained by reservoir conditions, deliverability of the pipeline network, fluid handling capacity of facilities, safety and economic considerations, or a combination of these considerations. The task of field operators is to devise optimal operating strategies to achieve certain operational goals. These goals can vary from field to field and with time. Typically one may wish to maximize daily oil rates or minimize production costs. This research aims to develop optimization methods that ease and automate the decision making of field operators for certain operations. In this section, the major components (the objective, the control variables, and the constraints) of the petroleum field optimization problem are described in detail.

*Corresponding Author: Mohammad Soleimani, Department of Petroleum Engineering, University of Tehran, Kish International campus, Iran (mmsoleimani1366@gmail.com)
Objective. Usually, the objective is to maximize the profit from an oil field on a day to day basis. How to define the profit is a complicated issue that requires intensive study in its own right. To keep this research focused, we restricted our attention to simple objective functions such as maximizing weighted daily flow rates. We emphasize that we only optimize production operations for a short-term period. The flow rate and other operational settings, once determined, are assumed to remain fixed during that period. However the optimization procedures developed in this study can be used repeatedly for long-term hydrocarbon recovery studies.

Control variables. The control variables are the production operation settings to be optimized. In this study, the control variables included the lift gas rates, the production rates, and the well connections to flow lines. Continuous gas-lift is a common artificial lift method used in the oil industry to improve well performance. The mechanism of gas-lift is fairly simple. Gas is injected into the tubing string to lighten the liquid column and increase the bottom hole pressure, which allows the reservoir to push more fluids into the wellbore. At the same time, increased flow rates in the tubing string and surface flow lines result in higher backpressure on the well and adjacent wells that share a common flow line. This in turn causes a reduction in well production rates. Therefore, lift gas has to be carefully allocated to achieve maximum efficiency. The production rate of a well is usually controlled by a choke. Adjusting production rates is the most straightforward way to meet certain production targets and satisfy certain operational constraints as described later. In some petroleum fields such as the Prudhoe Bay oil field in Alaska, USA, a production well can be connected to different flow lines that lead to different separation units. In such fields, switching a well from one flow line to another flow line can be an effective way to relieve the delivering/processing burden of one device/facility and increase the delivering/processing efficiency of the overall system. Thus, well connections were also considered as one type of decision variables in this study.

Constraints. Production operations in an oil field are usually subject to multiple capacity, safety, and economic constraints.

Capacity constraints. It is not surprising to know that oil production in some petroleum fields is constrained by the processing capacities of surface facilities, such as the gas and water processing capacities of separators, and the gas compression capacity of a central gas plant. For example, oil production in the Prudhoe Bay oil field (Barnes et al., 1990) and the Kuparuk River field in Alaska (Stoisits et al., 1992, 1994, 1999) is constrained by the gas handling capacities of surface facilities. The main reason for this is that production systems are usually designed and installed at the very early stage of reservoir development when information about the reservoir and future economics are scarce or uncertain. Thus facility capacities may not always be able to meet the production demand throughout the life of a reservoir. These constraints can be satisfied and/or fully utilized by adjusting the lift gas and/or production rates and switching well connections between flow lines.

Safety and economic constraints. For safety reasons, we may need a maximum/minimum pressure constraint at the bottom of a well or on some surface facility nodes. To operate the production system economically, we may put a maximum/minimum flow rate constraints on certain production wells or facilities. Furthermore, corrosion/erosion can lead to costly repairs. To avoid excessive corrosion/erosion, fluid velocities may have to be limited (Svedeman and Arnold, 1994, Kermani and Harrop, 1996).

The problem of interest requires simultaneous allocation of production rates, lift gas rates, and well connections to flow lines. However, as discussed in Chapter 2, robust procedures for such a task are not available. Either previous investigators have addressed only a part of the problem, or ad hoc rules have been used that may lead to suboptimal operations. Hence, it was necessary to develop approaches that optimize all control variables simultaneously subject to all constraints. This dissertation investigated such approaches. The target problem is a nonlinearly constrained optimization problem with both continuous and discrete decision variables. A two-level programming approach was developed to solve the problem. In this approach, optimization is conducted in two levels. The upper level explicitly optimizes the well connections. For each set of well connections, the upper level spawns a lower level problem to find the optimal production and lift gas rates for that set of well connections. The upper level masters the overall optimization procedure and the lower level can be viewed as a function evaluation procedure for the upper level. The advantage of this two-level programming approach is flexibility. Because the continuous and discrete variables are optimized at different levels, the approach offers us more freedom to use existing methods or develop new methods for solving the overall problem.

Literature Review

Applications of optimization techniques in the upstream oil industry began in the early 1950s and have been flourishing since then. Applications have been reported for recovery processes, planning, history matching, well placement and operation, drilling, facility design and operation and so on. Optimization techniques employed in these applications cover almost all subfields in mathematical programming, such as linear programming,
integer programming, and nonlinear programming. In this chapter, we first introduce concepts, solution algorithms, and applications of some major subfields in mathematical programming. Then we review their applications in areas that are pertinent to this study. Mathematical programming is a field born in the later 1940s (Lenstra et al., 1991). Despite its short history, mathematical programming has developed into a sophisticated field with deep specialization and great diversification. Mathematical programming encompasses subfields such as linear programming, integer programming, nonlinear programming, combinatorial optimization, stochastic programming, and so on. Optimization problems in the most general form can be represented as

$$\min \{ f(x) : l_i \leq c_i(x) \leq u_i, i = 1, \ldots, m \} \tag{2.1}$$

where the objective function $f$ and the constraint functions $c_i$ are functions of control variable $x$, and $l_i$ and $u_i$ are the lower and upper bounds for the $i$th constraint, respectively. An optimization problem can be categorized according to the type of its control variables, and objective and constraint functions.

### Linear Programming

When the objective function $f$ and constraint functions are linear functions of control variable $x$, the problem described by Eq. 2.1 is a linear programming (LP) problem. Various kinds of optimization problems can be formulated as LP problems, such as transportation, production planning, resource allocation, and scheduling problems. In 1947, Dantzig proposed the first solution method for the LP problem, the simplex algorithm. The simplex algorithm (Dantzig, 1963) finds the optimal solution by moving along the vertices of the feasible region, thus its optimal solution is always a vertex (or an extreme point) of the feasible region. In general, the simplex algorithm is very efficient. However, for some problems, the simplex method can take an very large number of iterations (Klee and Minty, 1972). Karmarkar (1984) introduced the first interior point algorithm for the LP problem. The interior point algorithm approaches the optimal solution from the interior of the feasible region. Thus the solution usually is not a vertex of the feasible region. The interior point method is very efficient both theoretically and in practice. Nowadays, LP problems with thousands or even millions of variables and constraints can be solved efficiently by both the simplex and interior point algorithm (Bertsimas and Tsitsiklis, 1997).

### Integer Programming

When all components of the unknown $x$ are discrete variables, the problem described by Eq. 2.1 becomes an integer programming (IP) problem. When some but not all components of $x$ are discrete, the problem is a mixed integer programming (MIP) problem. Discrete variables are useful to model indivisibility, logical requirements, and on/off decisions. Integer programming finds its application in many real life problems, such as capital budgeting, airline crew scheduling, telecommunications, and production planning. For the production optimization problem addressed in this study, the well connections have to be modeled as discrete variables. For linear integer programming (LIP) problems, the common solution techniques are the cutting plane method and the branch and bound method. Both methods tackle the LIP problem by solving a series of linear programming problems. The cutting plane method was proposed by Gomory (1958). The fundamental idea is to add constraints to a series of linear programming relaxations of the LIP problem until the optimal solution of a relaxation problem takes integer values (a linear programming relaxation is formed by allowing the integer variables in an LIP problem to take real values). In practice, the cutting plane method is not effective. The most effective technique for solving the LIP problem is the Branch and Bound method, which was first presented by Land and Doig (1960). The Branch and Bound method uses a “divide and conquer” approach to explore the set of feasible integer solutions. The method divides an optimization problem recursively into multiple subproblems. Instead of enumerating all the subproblems, the method uses bounds on the optimal objective value of a subproblem to avoid forming and solving other subproblems. Other methods for integer programming include dynamic programming (Bellman, 1957), simulated annealing (Kirkpatrick et al., 1983), genetic algorithms (Goldberg, 1989), and many special algorithms designed for particular problems. So far, there does not exist a universal algorithm for integer programming: either the method takes a large amount of time, or it only gives an approximate solution (Bertsimas and Tsitsiklis, 1997).

### Nonlinear Programming

Eq. 2.1 becomes a nonlinear programming problem when its objective and/or constraint functions are nonlinear. Nonlinear programming problems come in many different forms and shapes. Algorithms have been developed for individual classes of problems. It is impossible to give a thorough survey of all major optimization algorithms in a limited space. Rather, this section gives a brief description of some of the optimization methods encountered most frequently in the petroleum engineering literature. Some general references about nonlinear programming are Gill et al. (1981), Fletcher (1987), and Bertsekas (1982).

### Unconstrained optimization methods

An important class of methods for unconstrained optimization problems is the so-called line search. Line search methods approach a local minimum using the following iteration scheme:

$$x_{k+1} = x_k + \alpha_k p_k \tag{2.2}$$
where \( x_k \) and \( x_{k+1} \) are the current and next iterates, \( p_k \)
is a search direction along which the function decreases, and \( \alpha_k \) is a step length that ensures “sufficient” progress toward the solution.

A Newton-based line search method (usually termed as Newton’s method) goes as follows. At each iteration, the method approximates the objective function \( f(x) \) around current iterate \( k \) using a quadratic function defined as follows:

\[
g_k(p) = f(x_k) + (x_k)^T p + \frac{1}{2} p^T \nabla^2 f(x_k) p
\]

When the Hessian matrix, \( \nabla^2 f(x_k) \), is positive-definite, the model function \( g_k(p) \) has a unique minimum which can be obtained by solving the following linear system

\[
\nabla^2 f(x_k) p_k = -\nabla f(x_k) \tag{2.4}
\]

The solution serves as the search direction of current iteration. The step length can be obtained by a linear search procedure. Newton’s method converges quadratically near a local optimum (Gill et al., 1981). The key to this exceptional convergence rate is that the method uses the curvature information of the objective function (the Hessian matrix) to compute the search direction. However, sometimes it is impossible or time-consuming to compute the Hessian matrix. Quasi-Newton methods are based on the idea of gradually building up an approximate Hessian matrix using the gradient information collected from previous iterations.

**Constrained optimization methods.** The optimum of a constrained optimization problem is characterized by a certain set of conditions. These conditions were first established by Kuhn and Tucker (1951) and they are often called the Kuhn-Tucker conditions or optimality conditions. Constrained optimization methods are developed to find a point that satisfies these conditions. The major constrained optimization methods include sequential quadratic and linear programming methods, reduced-gradient methods, and methods based on augmented Lagrangians, penalty, and barrier functions (Gill et al., 1981). The sequential quadratic programming (SQP) methods are widely regarded (Murray, 1997) as the most effective methods for nonlinearly constrained programming (NCP) problem. The SQP methods have a structure of major and minor iterations. Each major iteration involves formulating a quadratic programming (QP) subproblem to obtain the search direction \( p \) and using a line search procedure to obtain the steplength \( \alpha_k \). The QP subproblem is formulated in such a way that its objective function is a quadratic approximation to the Lagrangian function of the original problem and its constraint functions are the linearization of the original nonlinear constraints around the current iterate. The minor iterations are used to solve a particular QP subproblem. The sequential linearly constrained (SLC) methods (Murtagh and Saunders, 1982) are another class of methods widely used for NCP problems. Similar to the SQP methods, SLC methods also involve major and minor iterations. In contrast to SQP methods, which formulate a QP subproblem in each major iteration, the SLC methods formulate a linearly constrained subproblem whose objective function is a general approximation to the Lagrangian function of the original problem. In general, an SLC method tends to require fewer major iterations but more evaluations of the problem functions than an SQP method (Murray, 2000). The barrier methods solve a nonlinearly constrained problem by solving a sequence of unconstrained optimization problems. A barrier method starts with a feasible point and creates a sequence of unconstrained problems whose successive minimum stays feasible and converges to a minimum of the constrained problem. To achieve this, the objective function of the unconstrained problem is constructed by adding a barrier function to the original objective function of the constrained problem. The barrier function is a modification of the constraint functions and becomes infinitely large when the iterate approaches the boundary of the feasible region. The barrier method was first established in the 1960s for general nonlinearly constrained optimization problems (Fiacco and McCormick, 1968). Recently the method has been developed further to address optimization problems whose objective and constraint functions are convex functions (Boyd and Vandenberghe, 2001).

**Applications of Optimization Techniques to Petroleum Fields**

Optimization techniques have been applied to virtually all aspects of the oil industry. In this section, we review applications of optimization techniques in rate allocation, production system design and operations, and reservoir development and management.

**Lift gas allocation.** The mechanism of gas-lift is elaborate. An appropriate amount of lift gas increases the oil rate, while excessive lift gas injection will reduce the oil rate. To determine the optimal lift gas rate, the usual practice is to allocate the lift gas to a well according to a gas-lift performance curve (Nishikiori et al., 1989). A gas-lift performance curve is a plot of oil rate versus lift gas rate for a gas-lift well. When the gas supply is unlimited, the optimal lift gas rate is the one corresponding to the maximum oil rate on the performance curve. When the gas supply is limited, the lift gas is usually allocated using some optimization algorithm. The earliest lift gas allocation method is a simple heuristic method based on the concept of equal-slope, which states that at the optimal solution, the slope of the gas-lift performance curves should be equal for all wells (Hong, 1975; Kanu et al., 1981).

**Options in commercial simulators.** In some commercial reservoir simulators (GeoQuest, 2000;
Field cases. Rate allocation problems are encountered frequently in mature fields where production facilities cannot meet the field demand. For example, oil production in the Prudhoe Bay oil field in Alaska is constrained by the gas handling capacity of the separation units and the central gas plant. To utilize existing facilities fully, Barnes et al. (1990) developed the Western Production Optimization Model (WPOM) for the Prudhoe Bay field.

2. MATERIAL AND METHODS

A typical oil field contains a gathering system, a fluid distribution network, and an injection network. The gathering system collects the fluids from production wells and delivers them into separation units. The separated fluids are then distributed to different destinations for storage, sale, disposal, injection, or further processing. The injection network is used to inject fluids into the reservoir for enhanced oil recovery projects or for fluid disposal/storage. The multiphase flow problems in the gathering, distribution, and injection network belong to network flow problems that have special properties. For the production optimization problem considered in this work, a model is required to simulate the multiphase flow in the gathering system. The distribution and injection networks are ignored. The network problem with single-phase flow is relatively easy to solve and has been studied extensively in both the civil engineering and the petroleum engineering literature. There are two major solution approaches. Both approaches formulate the network problem according to the requirements of mass balance and Kirchoff's law. The first approach is to formulate the network problem as a set of system equations and solve it using the Newton-Raphson method or one of its variations (Donachie, 1973; Epp and Fowler, 1970; Liu, 1969). The second approach is to formulate the network problem as an optimization problem and solve it using optimization algorithms (Hall, 1976; Collins et al. 1978).

In this work, a homogenous model with slip was used to model multiphase pipe flow. In the homogeneous model, different phases are lumped into one single pseudophase so that pressure drop equations for single-phase flow can be utilized. For the pseudophase, mixture density \( \rho_m \) is based on in situ phase fractions:

\[
\rho_m = \rho_l E_l + \rho_g E_g
\]

where \( E_l \) and \( E_g \) are the liquid and gas phase in situ fractions, respectively, and

\[
E_l + E_g = 1
\]

Computation of gas volume fraction \( E_g \) is presented in Section 3.3.2. Ouyang (1998) suggested that the mixture velocity should be defined as

\[
U_m = \frac{\rho_l}{\rho_m} U_l + \frac{\rho_g}{\rho_m} U_g
\]

where \( U_l \) and \( U_g \) are the superficial velocities of liquid and gas phases, respectively. With these properties defined, we can proceed to compute the pressure loss for multiphase flow in pipes.
Rate Allocation through Separable Programming

Rate allocation refers to the problem of optimally adjusting production rates and lift gas rates of production wells to achieve certain operational goals. These goals vary with the field and time. In some petroleum fields, especially mature fields, oil production can be constrained by fluid handling capacities of facilities. For such fields, rate allocation can be an effective way to increase the oil rate or reduce the production cost. For example, if the oil production in a field is constrained by the gas processing capacity of the separation units, closing or reducing production rates of wells with the highest GOR will increase the total oil rate; in addition, reducing the lift gas rate of certain wells may increase the overall oil production by utilizing the gas processing capacity more efficiently.

This study addressed the following rate allocation problem. The objective function is the total oil rate. An oil, gas, water, or liquid flow rate constraint can be put on any production well or network node. In abstract form, the optimization problem can be expressed as

\[
\text{maximize } \sum_{r=1}^{n_r} \left\{ q_{x,j}^{r} \right\}
\]

subject to

\[
d_{p,j}^{n} \leq q_{p,j}^{n}, \quad j \in \Omega^{n}, \quad p \in \{o, g, w, f\}
\]

where \( q_{x,i}^{r} \), is the oil rate of well \( i \), \( q_{p,j}^{n} \), is the flow rate of phase \( p \) in node \( j \), \( q_{p,x}^{n} \), is the flow rate limit of phase \( p \) for node \( j \), \( \Omega^{n} \) is the set of all nodes. For gathering systems with a tree-like structure, the flow rate of network node \( j \) is the sum of the flow rates of wells connected to node \( j \). Physically, the control variables for the rate allocation problem are the well chokes and lift gas rates of wells. And the flow rate of each well, \( q_{x,i}^{r} \), is a nonlinear function of the control variables. Fang and Lo (1996) studied a similar rate allocation problem. To simplify the optimization problem, Fang and Lo (implicitly or explicitly) made the following assumptions:

1. The well performance information can be evaluated individually for each well by ignoring flow interactions among wells.
2. The GOR and water cut for a well remain constant for varying oil rate.
3. The gas-lift performance curves are concave.

With the above assumptions, Fang and Lo (1996) reformulated the rate allocation problem to a linear programming problem and solved it by the simplex algorithm. The method was found to be very efficient.

In this part of the work we followed the line of Fang and Lo (1996) and developed other solution methods for the rate allocation problem. It was our aim to develop techniques that speed up the entire optimization process. In Chapter 5, we will present a method that does not rely on assumption 1.

Well Performance Estimation

The method presented in this chapter required the performance estimation of individual wells. Specifically, for unlifted wells, the method requires (1) the maximum oil rate that is allowed or can be delivered from a well, and (2) the water and gas rates as functions of the oil rate of a well. For lifted wells, the method requires (1) the oil rate as a function of the lift gas rate (the gas-lift performance curve), and (2) the water and (formation) gas rates as functions of the oil rate of the well. The well performance information can be described by a set of oil rate versus water, formation gas, and lift gas rate curves (Figure 1), which are referred to in this study as the performance curves. How to construct the performance curves is an extensively studied area (Beggs, 1991; Lea and Tighe, 1983). Two common approaches are: (1) derive the performance curves from well test information, or (2) estimate the performance curves through simulation. This study adopted the second approach.

![Figure 1: Illustration of well performance curves.](image)
approximated by piecewise linear performance curves, the water, formation gas, and lift gas rates of a well can be regarded as functions of the oil rate of that well. Therefore, if we regard the oil rate as the control variables, denoted as $x$, Eq. 4.1 becomes an optimization problem whose objective and constraint functions are the sums of functions of one variable, which can be expressed as

$$\text{maximize} \sum_{j=1}^{n_w} f_j(x_j)$$

subject to $\sum_{j} h_{iy}(x_j) \leq b_i$, $i = 1, \ldots, m$

$$x_j \geq 0, \quad j = 1, \ldots, n_w$$

where $f_j$ denotes the objective functions, $h_{iy}$ denotes the $j$th function involved in the $i$th constraint, $b_i$ denotes the limit of the $i$th constraint, and $m$ denotes the number of constraints. Optimization problems of the form of Eq. 4.2 are separable piecewise linear problems, which can be solved by linear optimization techniques (Gill et al., 1981).

Model MILP-I
This model was applied to a rate allocation problem by Güyagüler and Byer (2001). This model enforces constraint Eq. 4.5d explicitly and is suitable for rate allocation problems with performance curves of arbitrary shapes. The disadvantage of this method is that even for a rate allocation problem with moderate size, this model can contain a large number of binary variables and constraints.

Model MILP-II
This model was developed in this study. This model is motivated by and is suitable for problems with the following properties:
1. For wells that can flow naturally, their gas-lift performance curves are concave
2. For wells that need a finite amount of lift gas to start flowing, their gas-lift performance curves are comprised of a horizontal line and a concave curve
3. The oil versus water and formation gas curves are concave.

Model LP-II
This model was first proposed by Lo and Holden (1992). This is a simple model that does not require the performance curves and therefore there is no need to satisfy constraint Eq. 4.5e. The model assumes the well water cut and GOR are constant for varying oil rates.

DISCUSSIONS

Model LP-I and LP-II are linear programming (LP) problems. Model MILP-I and MILP-II are mixed integer linear programming (MILP) problems. These models are also called the LP-I, LP-II, MILP-I, and MILP-II methods in this study, respectively. All these methods require the objective and constraint functions to be in separable form, thus they are also referred to as the separable programming (SP) techniques.

CONCLUSIONS

This study addressed the problem of optimizing the production rates, lift gas rates, and well connections to flowlines subject to multiple flow rate and pressure constraints to achieve certain short-term operational goals. This problem is being faced in many mature fields and is an important element to consider in planning the development of a new field. In this research, we developed, compared, and tested optimization methods for petroleum production problems. The major contributions of this work and areas for further study are summarized in this section.

1. The optimization methods investigated and developed here are effective for problems of varying complexities and sizes. The methods can be used for both short-term production optimizations and long-term reservoir development studies. For instance, they can be used to optimize well connections and well rates simultaneously, even for large production systems such as those in the Prudhoe Bay field.
2. For the rate allocation problem, when flow interactions among wells are not significant, the various separable programming methods (the LP-I, LP-II, MILP-I, MILP-II methods described in Chapter 4) proved to be better than other conventional approaches used in the oil industry (i.e., the equal-slope method for gas-lift optimization, and the rule-based methods available in commercial simulators).

Specifically, the MILP-I method can be used for problems with performance curves of arbitrary shapes. The MILP-I, LP-I, and LP-II methods are suitable for large-scale rate allocation problems. The MILP-II method is especially suitable for gas-lift optimization problems with many non-concave gas-lift performance curves.
3. For the rate allocation problem, when flow interactions have significant impact on the optimal solution, simulation models capable of capturing such flow interactions should be used in the optimization process. Formulation P2 (Chapter 5) proved to be most appropriate for this problem. The formulated nonlinearly constrained problem (NCP) can be solved efficiently by a sequential quadratic programming (SQP) algorithm.
4. For the optimization of well connections, the partial enumeration (PE) algorithm proved to be both efficient and robust compared to a genetic algorithm (GA).
5. The two-level programming approach developed for the entire production optimization problem is flexible. Different optimization algorithms can be used for the upper level problem and the lower level problem.
6. For a production optimization problem with multiple objectives, the weighted-sum method may fail to obtain solutions of interest to us.
7. Multiple solutions may exist for the multiphase flow problem of a gathering system. This may bring computational difficulties to full field simulations and production optimizations. Post-analysis should be performed to ensure the validity of a solution from a simulation or an optimization run.

Summary of Contributions
The major contributions of this study can be summarized as follows:
1. Optimization methods were developed and tested on various synthetic examples and applied successfully to a real petroleum field, the giant Prudhoe Bay oil field in Alaska. Results demonstrated that the methods developed have distinct advantages over conventional production optimization approaches.
2. A two-level programming approach was developed to optimize the production rates, lift gas rates, and well connections simultaneously subject to multiple flow rate and pressure constraints.

REFERENCES


BIBLIOGRAPHY 158 DUTTA-ROY, K., BARIJA.